Name:.....

HSC Mathematics

Assessment Task 1 – 2013

Time Allowed – 60 minutes + 5 minutes reading

Write using black or blue pen. Black pen is preferred
Board-approved calculators may be used
In questions 5 – 7, show relevant mathematical reasoning and/or calculations
Answers to the multiple choice are to be done on the answer sheet provided.
The graph for question 7c is on the back of the multiple choice answer sheet.

Section 1	/4
Section 2: Derivative	/12
Section 2: Locus	/12
Section 2: Applications of Calculus	/12
Total	/40

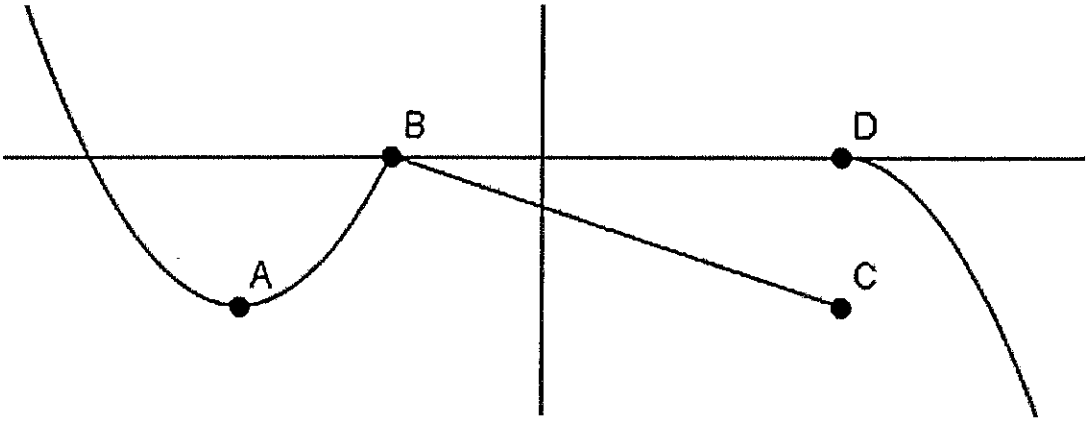
Section 1

4 marks

Allow about 6 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 4

1) At which point is the curve continuous but not differentiable 1



2) What is the locus of the moving point that is always 2 units from (3,1) 1

(A) $(x-3)^2+(y-1)^2=2$

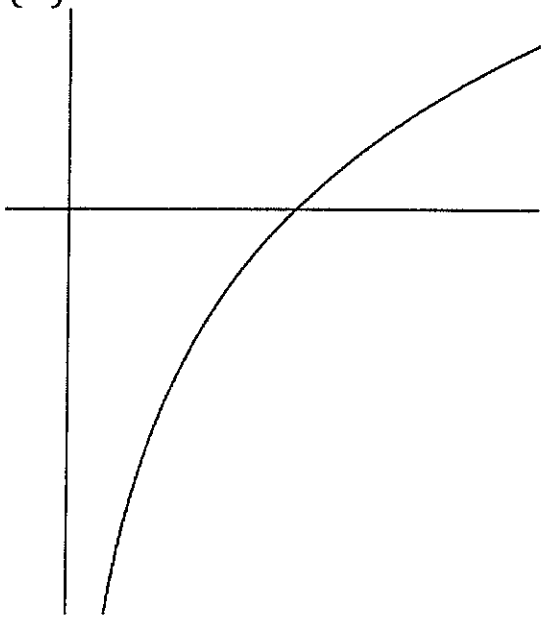
(B) $(x-3)^2+(y-1)^2=4$

(C) $(x+3)^2+(y+1)^2=2$

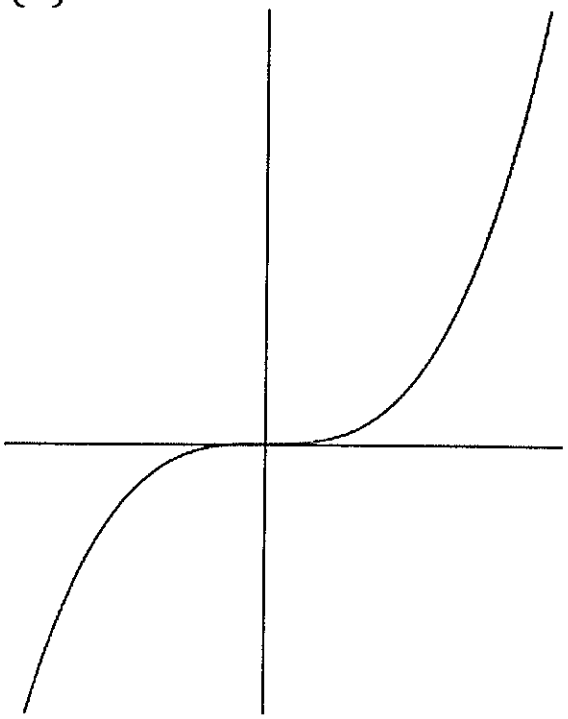
(D) $(x+3)^2+(y+1)^2=4$

3) Which of the following graphs has $f'(x) \neq 0$ and $f''(x) = 0$ in the domain shown. 1

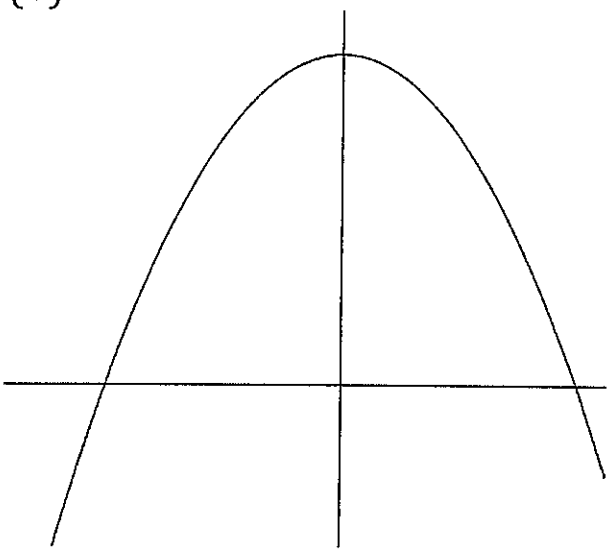
(A)



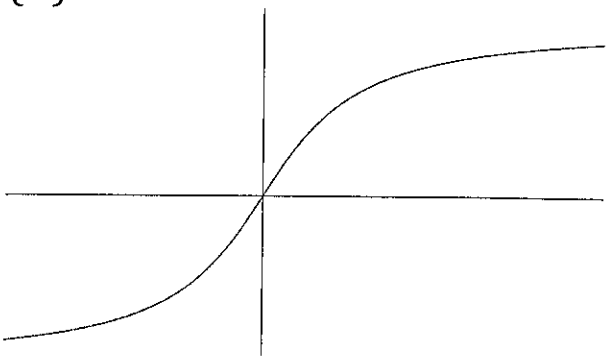
(B)



(C)

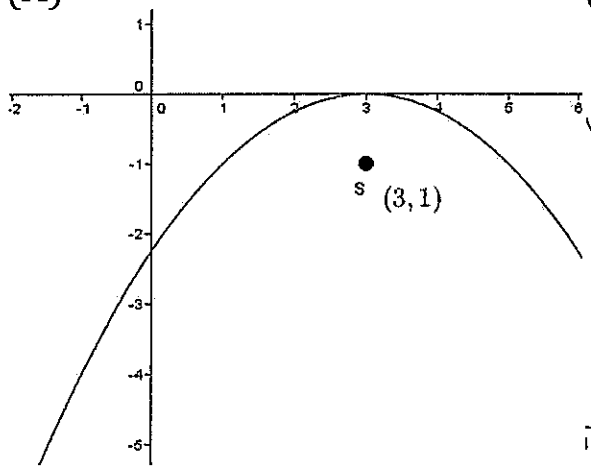


(D)

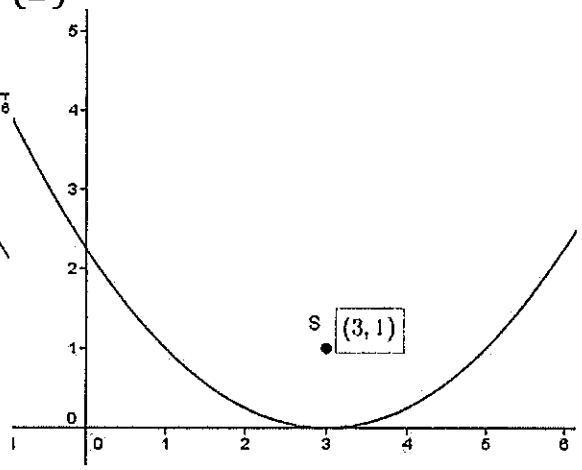


4) The parabola with equation $(3-x)^2 = 4y$ is shown in which picture 1

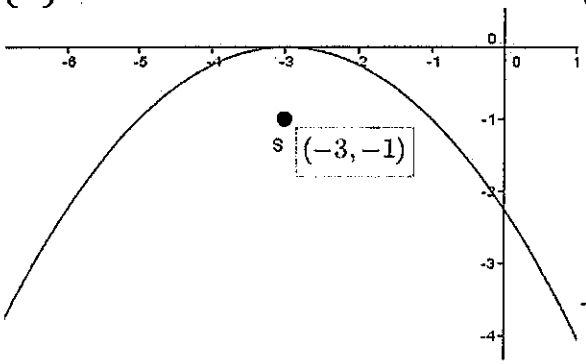
(A)



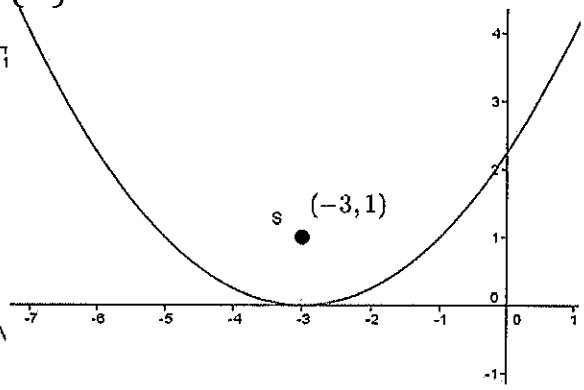
(B)



(C)



(D)



Section 2

36 marks
Allow about 54 minutes for this section

Question 5) **Use a new piece of paper**

a) Find the derivative of

i) $f(x) = x^3 + 4x + 5$ 1

ii) $f(x) = \frac{1}{x^4}$ 1

iii) $f(x) = \sqrt{x}$ 1

b) Find the derivative of

i) $y = \sqrt{1 - x^3}$ 1

ii) $y = x^3(1 + x)^5$ 2
(write your answer in simplified factored form)

iii) $y = \frac{2x-3}{1-x^2}$ 2

c) Find the equation of the tangent to $y = x^3 - 2x$ at the point $(-1,1)$ 2

d) The slope of the normal to the curve $y = 3 - ax^3$ at $x = 2$ is 1. Find the value of a. 2

Question 6) **Use a new piece of paper**

a) Write the locus of all points 4 units from the x −axis 1

b) i) Given that $A = (1,2)$ and $B = (-2,6)$, show that the locus of the point $P(x, y)$ which moves such that the line segments PA and PB are perpendicular is a circle with equation 3

$$x^2 + x + y^2 - 8y = -10$$

ii) By completing the square (or otherwise), find the centre and radius of the circle in part i) 2

c) Write the equation (DO NOT DERIVE) of the locus of a point moving so that it is equidistant from the line $y = -1$ and the point (2,3) 2

d) For the parabola with equation $(y - 3)^2 = 12(x - 2)$

i) Find the focal length 1

ii) Find the coordinates of the vertex 1

iii) Find the coordinates of the focus 1

iv) Find the equation of the directrix 1

Question 7) **Use a new piece of paper**

a) For the function $f(x) = 3x^2 + 2x^3$ find

i) The coordinates and nature of any stationary points 3

ii) The coordinates of any points of inflexion 2

iii) Sketch the curve showing all important features 3

b) On what domain is the function $f(x) = x^3 - 2x^2 + 2x + 1$ increasing 2

c) On the number plane provided on the back of the multiple choice answer sheet, draw the gradient function of the given graph 2

Mathematics

Student Name/Number:_____

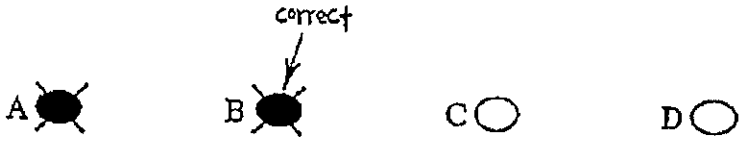
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 \div 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A○ B● C○ D○

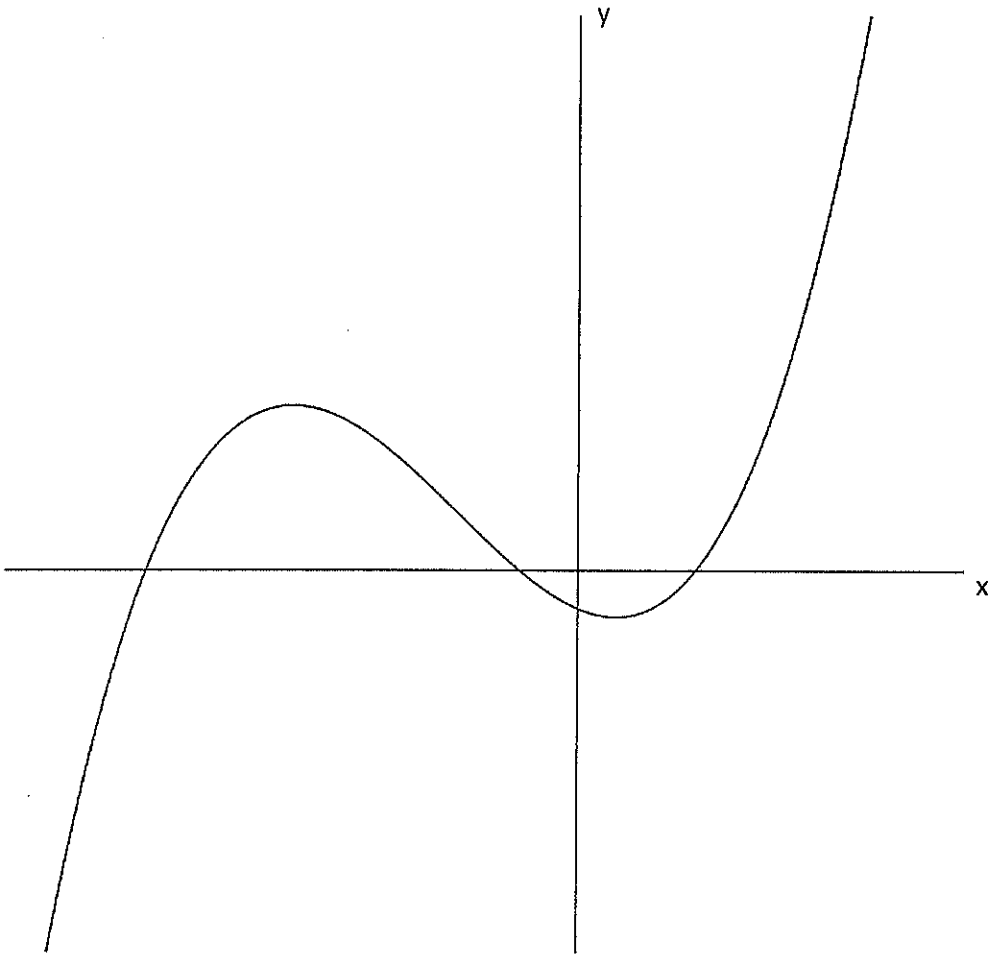
If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○

Question 7c



Mathematics

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Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A○ B● C○ D○

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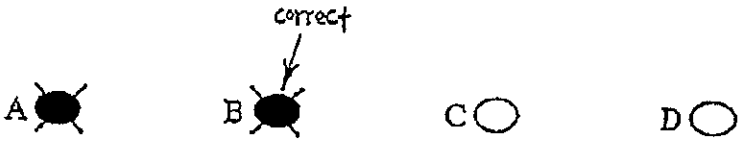
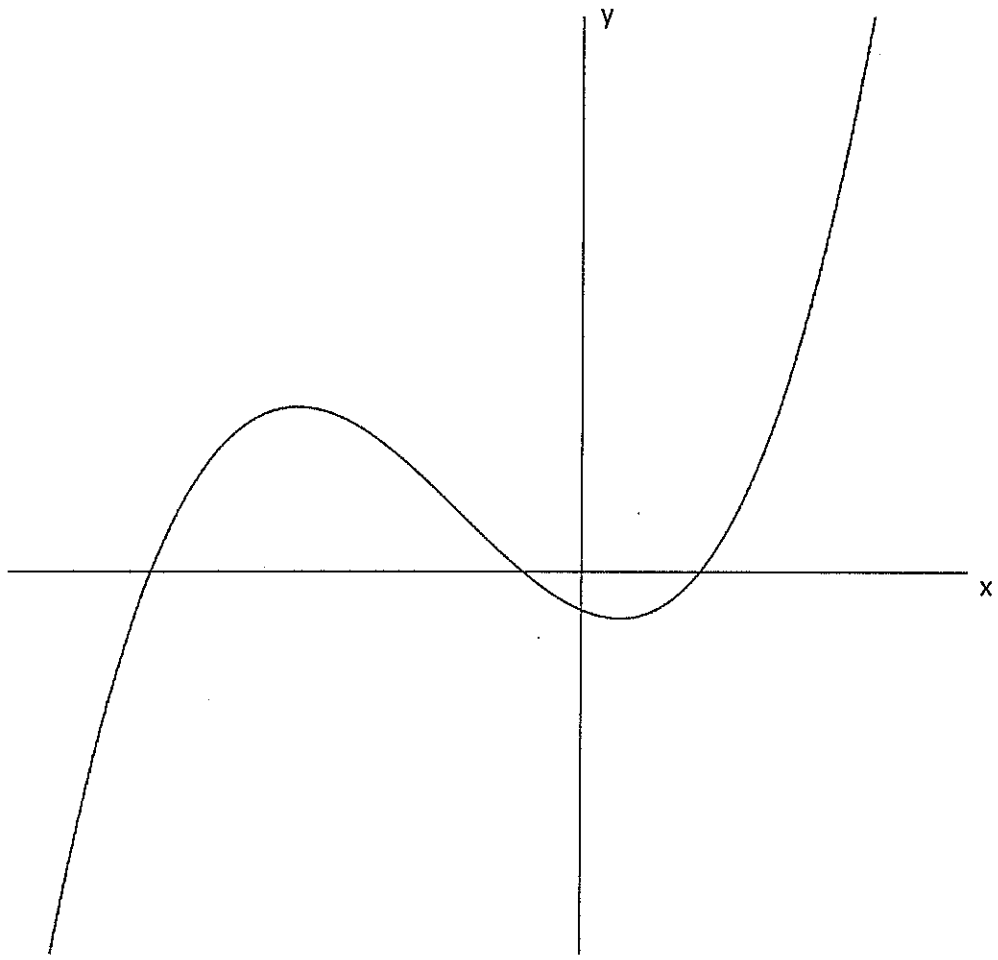


1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○

Question 7c



2013/14 HSC
Mathematics T1

Multi-choice

1) B
2) $(x-3)^2+(y-1)^2=4$ B
3) D
4) B

Question 5

a) i) $f'(x) = 3x^2+4$

ii) $f'(x) = -4x^{-5}$
$= -\frac{4}{x^5}$

iii) $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$
$= \frac{1}{2\sqrt{x}}$

b) i) $y' = \frac{1}{2}(1-x^3)^{-\frac{1}{2}} \times -3x^2$
$= -\frac{3x^2}{2\sqrt{1-x^3}}$

ii) $\frac{dy}{dx} = 3x^2(1+x)^5 + x^3 \times 5(1+x)^4 \times 1$
$= (1+x)^4 x^2(3(1+x)+5x)$
$= (1+x)^4 x^2(3+8x)$

iii) $y' = \frac{(1-x^2)\times 2 - (2x-3)\times -2x}{(1-x^2)^2}$
$= \frac{2-2x^2+4x^2-6x}{(1-x^2)^2}$
$= \frac{2-6x+2x^2}{(1-x^2)^2}$

c) $\frac{dy}{dx} = 3x^2 - 2$
@ $x=-1$ $m = 3-2$
$= 1$
$y-1 = 1(x+1)$
$y = x+2$

d) $m_{normal} = 1$
$m_{tangent} = -1$
$y' = -3ax^2$
$-3ax^2 = -1$ @ $x=2$
$-3\times a\times 4 = -1$
$a = \frac{1}{12}$

b) a) $y = \pm 4$

b) i) $m_{PA} = \frac{y-2}{x-1}$

$m_{PB} = \frac{y-6}{x+2}$

$\therefore \frac{y-2}{x-1} \times \frac{y-6}{x+2} = -1$

$\frac{y^2-8y+12}{x^2+x-2} = -1$

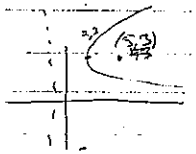
$y^2-8y+12 = -x^2-x+2$

$x^2+x+y^2-8y = -10$

ii) $x^2+x+\frac{1}{4}+y^2-8y+16 = -10+\frac{1}{4}+16$

$(x+\frac{1}{2})^2+(y-4)^2 = \frac{25}{4}$

c) $(x-2)^2 = 12(y-1)$



d) i) $a = 3$
ii) $(2,3)$
iii) $(5,3)$
iv) $x=-1$

7) a) i) $f'(x) = 6x+6x^2$
$= 6x(1+x)$

stat pts when $f'(x)=0$
$6x=0$ $1+x=0$
$x=0$ or $x=-1$
@ $x=0$ $y=0$ @ $x=-1$ $y=1$

$f''(x) = 6+12x$

@ $x=0$ $f''(x) > 0$
$\therefore (0,0)$ is a min turning pt.

@ $x=-1$ $f''(x) = -6$
< 0
$\therefore (-1,1)$ is a max tp.

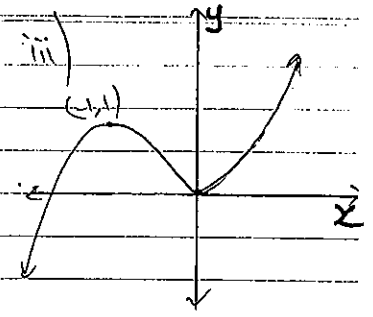
ii) potential points of inflexion when
$f''(x) = 0$
$6+12x = 0$
$x = -\frac{1}{2}$

test for change in concavity

x	-1	$-\frac{1}{2}$	0
$f''(x)$	+ve	0	+ve

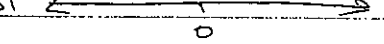
$\therefore$ concavity changes
$\therefore (-\frac{1}{2}, \frac{1}{2})$ is a point of inflex

iii)



b) $f'(x) = 3x^2-4x+2$
increasing when $f'(x) > 0$
$3x^2-4x+2 > 0$
consider equality.
$3x^2-4x+2 = 0$
$x = \frac{4\pm\sqrt{16-4\times3\times2}}{2\times3}$
$= \frac{4\pm\sqrt{-8}}{6}$
$\therefore$ no solns

Test



$\therefore$ increasing for all x

c)